International Journal of Electrical and Electronics Engineering (IJEEE)
ISSN(P): 2278-9944; ISSN(E): 2278-9952
Vol. 3, Issue 6, Nov 2014, 1-10

International Academy of Science, Engineering and Technology
Connecting Researchers; Nurturing Innovations

ALTERNATING DIRECTION METHOD OF MULTIPLIERS FOR BLIND IMAGE DEBLURRING WITH UNKNOWN BOUNDARIES

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ABSTRACT

Now days, deblurring plays a vital role in all image processing tasks. Blind image deblurring (BID), can be solved by imposing some form of regularization (prior knowledge) on the unknown blur and original image. Here we introduce a new version, in which both the optimization problems with respect to the unknown image and with respect to the unknown blur are solved by the alternating direction method of multipliers (ADMM) – an optimization tool that has recently sparked much interest for solving inverse problems, namely due to this efficiency and less complexity. This algorithm also gets better results or realistic case of blind deblurring with unknown boundary conditions. Experiments with synthetic and real blurred images show the competitiveness of the proposed method, both in terms of speed and restoration quality.

KEYWORDS: Alternating Direction Method of Multipliers (ADMM), Blind Deblurring, Blind Deconvolution, Non Blind Image Deconvolution (NBID)

1. INTRODUCTION

Blind image deblurring (BID) is an inverse problem where the observed image is modeled as resulting from the convolution with a blurring filter, possibly followed by additive noise[1], and the goal is to estimate both the underlying image and the blurring filter. Clearly, BID is a severely ill-posed problem, for which there are infinitely many solution. Furthermore, the convolution operator is itself typically ill-conditioned, making the inverse problem extremely sensitive to inaccurate filter estimates and to the presence of noise. To deal with the ill-posed nature of BID, most methods use prior information on the image and the blurring filter. Concerning the blur, earlier methods typically imposed hard constraints, whereas more recent ones use regularization. Those methods are thus of wider applicability, e.g., to the practically relevant case of a generic motion blur, typically addressed by encouraging sparsely of the blur filter estimate. This paper builds upon the method proposed in.

Which stands out for not using restrictions or regularizes on the blur (apart from a limited support), being able to recover a wide variety of filters. Due to the undetermined nature of BID, direct minimization of the cost functions typically used for deconvolution may not yield the desired sharp image estimates [2][3].

In fact, these sharp estimates typically correspond to local (not global) minima of these cost functions. Several strategies have been devised to address this Issue, such as the alternating estimation of the image and the blur filter, the use of restrictions, normalization steps, and careful initialization. Recently, a normalized image prior was proposed so that the global minimum would not correspond to the blurred image. Multi-resolution approaches, which avoid some local minima, can also be found by using continuation schemes, where the regularizing parameter is gradually decreased. In a Bayesian frame-work, it has been claimed that a MAP estimate of the blur filter (after marginalizing out the unknown image) is preferable to a joint MAP estimate of the image and the filter [3][4][5][6].

Most blind and non-blind deblurring methods assume periodic boundary conditions (to allow using FFT-based convolutions), instead of the more realistic unknown boundary conditions (UBC) [7]. This incorrect assumption is a problem in non-blind deblurring and becomes worse in BID (although it has mostly been ignored), since the filter estimate is affected by the inaccuracy of the cyclic model. A simple way to avoid the UBC problem is to use the ”edge taper” function, which softens the boundaries of the degraded images, reducing the effect of wrongly assuming periodic boundary conditions; this approach is used in, while [8] employs a more sophisticated version thereof [9]. Other works on BID [10], although not explicitly reporting it, adopt some strategy for dealing with the boundaries, since they present good results on real blurred images.

In this paper, we improve up on the method of deblurring. We fully embrace the UBC, without an increase in computational cost, due to the way in which we use the alternating direction method of multipliers (ADMM) to solve the minimizations required by that method. Using the ADMM, we also manage to impose positivity on the blurring filter, reaching considerable speed and quality improvements over the original version. The paper is organized as follows: section 2 sets the scenario, by introducing the BID problem, reviewing the method of, and the ADMM; section 3 introduces the proposed approach, and section 4 reports experimental results.

2. BACKGROUND

2.1. Observation Model

Consider the linear observation model y=Ax+n, where $y \in R^n, x \in R^m$ and $n \in R^n$ are vectors containing the pixels (lexico-graphically ordered) of the degraded image, the (unknown) original image, and the additive noise, respectively; $A = H \in R^{n \times m}$, where H is the matrix representing the convolution with a blurring filter h. For computational convenience, most methods assume this convolution to be cyclic/periodic, thus n=m and H is a (block) circulant matrix, which is diagonal zed by the discrete Fourier transform (DFT). However, in real-life, the convolution is not cyclic and to obtain ann×n blurred image one must Access to $\sqrt{m} \times \sqrt{m}$ = $(\sqrt{n} + 2l) \times (\sqrt{n} + 2l)$ pixels of the original image, assuming the blurring filter to have a (2l+1)×(2l+1) support. In this case, the observation operator A=MH $\in R_{m \times m}$ can be factored into the product of a cyclic convolution H$\in R_{m \times m}$ with an masking matrix M$\in \{0,1\}_{n \times m}$, excluding the boundary where the cyclic convolutions is invalid.

2. 2 Existing Method

The implementation of this rationale is based on measures of spectral whiteness to assess the fitness of the current estimates to the degradation model. Residual whiteness has been used for a long time to assess model accuracy, namely in modeling time series and dynamical systems [11]; more recent applications can be found in spectroscopy [12] and signal detection.

However, to the best of our knowledge, criteria based on residual whitness have not been used before in image deconvolution / deblurring. Our criteria are particularly suited to the BID method, where stopping and choosing the regularization parameter are one and the same thing. The results reported in this paper, show that, on a large set of synthetic experiments, the proposed criteria lead to an average decrease of 0.15 dB in ISNR2, compared to what is obtained by stopping the algorithm at the maximum ISNR (which of course, cannot be done in practice, as it requires the original image), outperforming in this sense both the DP and the measure of. We also show tests on color images and on various real blurred images; although with these images, no quantitative results can be reported, we believe the results can be (subjectively) considered good. We show that the proposed criteria are also suitable for adjusting the regularization parameter and stopping criterion of NBID methods. In particular, we report experiments with two recent algorithms, using different blurs and noise variances. In this scenario, our approach is shown to be adequate, but does not outperform SURE-based methods.

Algorithm 1: Continuation-Based BID	
1.	Set $h\wedge$ to the identity filter, $X\wedge$=y and $\lambda=\lambda_{0;}$ choose α<1.
2.	Repeat
3.	$X\wedge\leftarrow \arg\min_x C_\lambda(X,h\wedge)$
4.	$h\wedge\leftarrow \arg\min_h C_\lambda(X\wedge,h)$
5.	$\lambda\leftarrow\alpha\lambda$
6.	Until stopping criterion is satisfied.

2. 3 The BID Method

Following [2] (but in [2] the filter was not imposed to have positive entries), the image X and the blurring operator H (equivalently, the filter h) are estimated by minimizing the cost function

$$C_\lambda(X,h)=\frac{1}{2}\|y-MHX\|^2{}_2+\lambda\sum_{i=1}^{m}(\|F_iX\|_2)^q+\ell_S+(h), \tag{1}$$

Here $\lambda\sum_{i=1}^{m}(\|F_iX\|_2)^q = \phi(X)$

Where ℓ_{S+} is the indicator function of the set S+,

$$\ell_S+(u)=\{0\Leftarrow u\in S^+$$
$$\infty\Leftarrow u\notin S^+ \tag{2}$$

S+ is the set of filters with positive entries in a given support (this positivity constraint was not considered in, $\lambda>0$ is the regularization parameter, and $F_i\in R_{4\times m}$ is the matrix that corresponds to four directional (sobel-type) edge filters at pixel i, with $q\in[0,1]$. As shown in [2], good results are obtained by minimizing (1) alternatingly with respect to h and X, while slowly decreasing the regularization parameter λ (Algorithm 1). The rationale behind this continuation scheme is that, with large λ, the initial image estimates are piece-wise smooth with sharp edges, which allows improving the estimate of the filter h; this in turn will allow reducing the weight of the regularize, thus yielding a better image estimate, and so on. In [2], image estimate _X was obtained by gradient descent and the filter estimate _h by conjugate gradient (CG). Here, we show how these two steps can be more efficiently computed by the ADMM.

2. 4 The ADMM

The ADMM [13][14] has recently emerged as an efficient tool to address several imaging inverse problems [15][16] and is related to other methods, namely split-Bregman (SB) and Douglas-Rachford. Recently, SB was used for BID, under total variation and sparsely regularization [17]; however, those methods do not consider the realist case of non-circular blurring. Consider the general unconstrained minimization problem

$$\min_{Z \in R^d} \sum_{J=1}^{J} g^{(j)}(G^{(j)}Z),$$

Algorithm 2: ADMM

Algorithm 2: ADMM.

1. Set k=0, choose $\mu^{(j)} > 0, u_0^{(j)}$, and $d_0^{(j)}$, for j=1,.....J.
2. Repeat
3. $r_{k+1} \leftarrow \sum_{j=1}^{J} \mu^{(j)} (G^{(j)})^T (u_k^{(j)} + d_k^{(j)})$
4. $Z_{k+1} \leftarrow [\sum_{j=1}^{J} \mu^{(j)} (G^{(j)})^T G^{(j)}]^{-1} r_{k+1}$
5. For j=1 to J do
6. $u_{k+1}^{(j)} \leftarrow prox_{g^{(j)}/\mu^{(j)}} (G^{(j)} Z_{k+1} - d_k^{(j)})$
7. $d_{k+1}^{(j)} \leftarrow d_k^{(j)} - (G^{(j)} Z_{k+1} - u_{k+1}^{(j)})$
8. end
9. k$\leftarrow$ k+1
10. until stopping criterion is satisfied.

Where $G_{(j)} \in R_{aj} \times d$ are arbitrary matrices and $g_{(j)} : R_{aj} \rightarrow R$ are functions. An equivalent constrained formulation is $\min_{Z \in R^n, u^{(1)} \in R^{p1}, \ldots, u^{(J)} \in R^{pJ}} \sum_{j=1}^{J} g^{(j)}(u^{(j)})$ (3)

Subject to $u^{(j)} = G^{(j)}Z,$ for j=1,…J.

Where the u(j) are the splitting variables. The ADMM to solve (3) takes from the Algorithm 2, as shown in [16]. The challenging steps are those in lines 4 and 6. Line 6 involves the proximity operator (PO) of each g(j); recall that the PO of a function f, defined as

$$prox_f(v) = \arg\min_X (1/2) \|v - X\|^2{}_2 + f(X),$$

Has a closed form expression for several choices of f. Concerning line 4, it was shown in [15,16] that the required inversion can be efficiently obtained in several cases of interest, namely using the FFT and/or fast wavelet/frame transforms.

3. ROPOSED ALGORITHM

We propose using ADMM to tackle each of the inner minimizations in Algorithm 1 (lines 3 and 4), with $C_\lambda(X,h)$ as defined in (1). Of course, for q<1, the problem is non-convex, thus we have no theoretical convergence guarantees; however, as shown below, the empirical performance of the algorithm is very competitive.

3.1. Updating the Image Estimate

The image Estimate update problem of Algorithm 1 (line 3) can be written in the unconstrained formulation as

$$C_\lambda(X,h) = \frac{1}{2}\left\| y - MHX \right\|^2{}_2 + \lambda \sum_{i=1}^{m} \left(\left\| F_i X \right\|_2 \right)^q , \tag{4}$$

And in constrained formulation (3) by letting J=m+1, and

$$G^{(j)} = F_j,\ forj = 1,...m. \tag{5}$$

$$G^{(m+1)} = H, \tag{6}$$

$$g^{(j)}(u^{(j)}) = \lambda \left\| u^{(j)} \right\|^q{}_2,\ forj = 1,....m \tag{7}$$

$$g^{(m+1)}(u^{(m+1)}) = \frac{1}{2}\left\| y - M u^{(m+1)} \right\|^2{}_2 \tag{8}$$

The key steps of the resulting instance of Algorithm 2 are (as mentioned above) those in lines 4 and 6. Line 4 can be written a

$$Z_{k+1} \leftarrow K(\rho H^T (u_k{}^{(J)} + d_k{}^{(J)}) + \mu \sum_{j=1}^{m} F_j{}^T (u_k{}^{(j)} + d_k{}^{(j)})),$$

Where we have set $\mu^{(1)} = ... = \mu^{(m)} = \mu$ and $\mu^{(m+1)} = \rho,$ and

$$K = (\rho H^T H + \mu \sum_{j=1}^{m} F_i{}^T F_i)^{-1} \tag{9}$$

If the convolutions with the edge filters represented by the matrices F_i are performed with periodic boundary conditions, K can be efficiently computed in the DFT domain (using the FFT), since both H and F are block-circulant matrices [15][16]

To implement line 6 of Algorithm 2, we need the two PO:

$$prox_{g^{(j)}/\mu}(v) = \arg\min_X \frac{\lambda}{\mu}\|X\|_2^{\ q} + \frac{1}{2}\|v - X\|^2_{\ 2}$$
$$= v - shrink(v, \lambda/\mu, q), ...(10)$$

For j=1….m, and (with I denoting an identity matrix)

$$prox_{g^{(m+1)}/\rho}(v) = \arg\min_X \frac{1}{\rho}\|y - MX\|_2^{\ 2} + \frac{1}{2}\|v - X\|_2^{\ 2} = \rho I + M^T M)^{-1}(M^T y + \rho v). \quad (11)$$

The proximity operator in (11) can be easily computed: M TM is a binary diagonal matrix, with zeros corresponding to the unobserved boundary pixels, and M Ty is the extension of $y \in R_n to R_m$ by zero-padding. Finally, "v-shrink" in (10) is a vectorial shrinkage function, which can be shown (details are omitted) to be given by

$$v - shrink(y, \tau, q) = \{y shrink(1, \tau\|y\|_2^{\ q-2}, q) if \|y\|_2 \neq 0$$
$$0 if \|y\|_2 = 0$$

Where shrink(z, τ ,q) = arg $\min_x$ 1 2 _Z- x_22+ $\tau|x|_q$ has closed form solutions for q∈ {0,1/2,2/3,1,4/3,3/2,2} (in some cases as functions of the roots of cubic and quadric equations[12]).

3.2 Updating the Blurestimate

The blur estimate update problem of Algorithm 1 (line 4) can be written in unconstrained formulation as

$$\min_h \frac{1}{2}\|y - MXh\|_2^{\ 2} + \ell S + (h),$$

And in constrained form(3), with J=2, $G_{(1)} = \mathbf{X}, G_{(2)} = I$, and

Where h$\in R_m$ is the vector containing the blurring filter elements (lexicographically ordered) and $X \in R_{m\times m}$ is the square matrix representing the convolution of image X with the filter in h. The resulting instance of Algorithm 2 involves (in line 4) the inversion of the matrix $\mu(1)XTX + \mu(2)I$ which can be efficiently computed in the DFT domain, using the FFT. Concerning the two (J=2) proximity operators in line 6, we have that $prox_{g(1)/\mu(1)}$ has exactly the same form as (11), with μ replacing ρ . Finally, since the proximity operator of the indicator of a convex set is imply the orthogonal projection on that set [18]

$$prox_{g^{(2)}/\mu^{(2)}}(v) = prox_{\ell S+}(v) = P_S + (v), ..(14)$$

Which consists in setting to zero any negative entries and those outside the given support?

4. EXPERIMENTS

In all the experiments, we use q=1/2, λ_0 =0.5,α=1/2, and the following setting for the two ADMM algorithms: (a) the image estimate (line 3 of Algorithm 1) is computed with 20 iterations of the algorithm explained in subsection 3.1, initialized with $d^{(j)} = 0,\ u^{(j)} = G^{(j)}X$ (where X is the estimate from the previous outer iteration of Algorithm 1), μ=0.5, and ρ =2λ; (b) the filter estimate (line 4 0f Algorithm 1) is computed with 15 iterations of the algorithm explained in subsection 3.2, initialized with $d^{(j)} = 0,\ u^{(j)} = G^{(j)}h$ (where h is the filter estimate from the previous outer iteration), $\mu^{(1)}$ =0.01, and $\mu^{(2)}$ =0.1; (c) all the ADMM penalty parameters $(\mu^{(j)})$ are updated using the empirical rule described in [13]. Both the proposed method and the method of [2] (implemented in MATLAB and run on an Intel Core i3 CPU) were stopped at the best ISNR (improvement in signal to noise ratio), in the synthetic experiments, or at the best visual result, for the real images.

The proposed approach was compared against its ancestor [2], in a set of 30 synthetic experiments with two benchmark images (Lena and Cameraman), five 9$\times$9 blur kernels (see figure 1) at three noise levels (BSNR $\in$ {∞, 40, 30} dB). Instead of periodic boundary conditions, we extended the images with values equal to the nearest boundary and both methods were run assuming unknown boundaries (see subsection 2.1). For most experiments, the proposed method led to considerably higher ISNR, while being more than three times faster; even higher speed-ups are expected if the fixed number of iterations is replaced by adequate stopping criteria. The average ISNR and processing times in Table 1 show that the proposed method clearly outperforms the baseline form [2].

Table 1: Comparison between the Baseline Method [2] and Our ADMM Approach. The Results for Each BSNR Value are Averages over the Five Blurring Filters and Two Images (Lena and Cameraman)

ISNR(dB)			Time(s)	
BSNR(dB)	[4]	proposed	[4]	proposed
∞	5.83	8.87	249	69
40dB	4.95	6.65	131	55
30dB	3.83	5.01	110	46

Experiment 5, Condition Number: 67.1

Experiment 7, Condition Number: 2.34×10^{7}

Figure 1: Deblurred Images in Different Experiments, Based on Three Criteria. From Left to Right: Highest ISNR, P-GSURE MAD, DP MAD, M RW

With their parameters manually adjusted for the visually best result. Besides being considerably faster, our method attained the best restoration, yielding an image with sharp edges and no significant articrafts. Figure 2**:** shows results obtained with an actual photo out-of-focus using the proposed approach and its ancestor method [2]. Our method attained a sharper image within one third of the processing time.

5. CONCLUSIONS AND ONGOING WORK

We have proposed a new algorithm for blind de convolution, improve-ing over the recent method of in two ways: a significant speedup (by using the ADMM) and the ability to handle unknown boundary conditions (more realistic than the usual periodic ones). Experiments with synthetic and real blurred images show that our method outperforms several state-of-art methods, both in terms of speed and restoration quality. Ongoing research aims at developing adequate stopping criteria for the inner ADMM algorithms, as well as for the outer iterations, namely following our recent work in.

REFERENCES

1. M. Almeida, M. Figueiredo, “Newstopping criteria foriterative Blindimage deblurring based on residual whiteness measures, ”in *IEEE Statist.Sig.Proc.Worksh*, 2011, pp. 337–340.
2. M. Almeida, L. Almeida, “Blindandsemi-blinddeblurringof Naturalimages,” *IEEE Trans.Image Proc.*, vol. 19, pp. 36–52, 2010.
3. A. Levin, Y. Weiss, F. Durand, W. Freeman, “Understanding and evaluating blind deconvolution algorithms,” in *IEEECVPR*, 2009.
4. R. Fergus, B. Singh, A. Hertzmann, S. Roweis,W. Freeman, “Removing camera shake from a single photograph,” *ACM Trans. Graph.*, vol. 25,pp.787–794,2006.
5. “Efficient marginal likelihood optimization in blind deconvolution,” in*IEEECVPR*, 2011.
6. J. Miskin, D. MacKay, “Ensemble learning for blind image Separation and deconvolution,” in *Proc. Int. Worksh. Independ. Comp. Analysis and Blind Source Separation–ICA*, 2000.
7. M. Almeida, M. Figueiredo, “Deconvolving images with unknown Boundaries using the alternating direction method of Multipliers,” *IEEE Trans. Image Proc.*, vol. 22, 2013.

8. Q. Shan, J. Jia, A. Agarwala, “High-quality motion deblurring from a single image,” *ACM Trans. Graph.*, vol.27, 2008.

9. R. Liu, J. Jia, “Reducing boundary artifacts in image deconvolution,” in *IEEEICIP*, 2008.

10. D. Krishnan, T. Tay, R. Fergus, “Blind deconvolution using a Normalized sparsity measure,” in *IEEECVPR*, 2011.

11. S. Cho, S. Lee, “Fast Motion Deblurring,” *ACM Transactions on Graphics (SIGGRAPH ASIA 2009)*,vol. 28,2009.

12. W. Li, Q. Li, W. Gong, S. Tang, “Total variation blind deconvolution EmployingsplitBregmaniteration,” *J. Vis. Common. Image Represent.*vol.23, pp. 409–417, 2012.

13. S. Boyd, N. Parikh, E. Chu, B. Peleato, dJ. Eckstein, “Distributed Optimization and statistical learning via the alternating Direction method of multipliers,” *Foundations and Trends in Machine Learning*, vol. 3, pp.1–122, 2011.

14. D. Gabay,B. Mercier,“Adual algorithm for the solution of Nonlinear variation problems via infinite element approximations,” *Comp.and Math. with Appl.*,vol.2, pp. 17–40, 1976.

15. M. Afonso, J. Bivouacs-Dias, M. Figueiredo, “Fast image recovery Using variable splitting and constrained optimization,”*IEEE Trans. Image Proc.*, vol. 19, pp. 2345–2356, 2010.

16. “Anaugmented Lagrangian approach to the constrained Optimization formulation of imaging inverse problems,” *IEEE Trans. Image Proc.*, vol. 20, pp. 681–695, 2011.

17. J.-F. Cai, H. Ji, C. Liu, Z. Shen,“Framelet-based blind motion Deblurring from a single image, ”*IEEE Trans. Image Proc.*, vol.21, pp. 562–572,2012.

18. P. Combettes, J.-C. Pesquet, “Proximal splitting methods in Signal processing,” in *Fixed-Point Algorithms for Inverse Problems in Science and Engineering*, H. Bauschke *etal*, Editors, Springer, 2011, pp.185–212.